Lesson Plan One

Discipline: Mathematics - Geometry	Subject Matter Focus: Triangle Theorems
Model of Teaching: Guided Discovery	Mathematical Practices: Model with mathematics (4), Look for and make use of structure (7)

Central Focus: The purpose of this learning segment is for students to build upon their knowledge of geometric theorems involving triangles. They will prove these theorems by constructing triangles using rulers and protractors, building and solving inequalities, and summarizing their conclusions with formal syntax and vocabulary. They will apply these theorems to solve mathematical problems and real-world situations.
Essential Question: How can we use the properties of triangles to solve real-world problems?

CCSS Standard(s): CCSSM.G-CO.10
Prove Geometric Theorems: Prove theorems about triangles.
ELD Standard(s): CCSS.ELD.9-10.S2.P1c.BR10a
Writing: Write clear and coherent summaries of texts and experiences by using complete and concise sentences and key words (e.g., from notes or graphic organizers)

Objective(s): By the end of this lesson, students will be able to explain why the Triangle Inequality Theorem is true for all triangles. They will be able to calculate the maximum and minimum lengths, in whole numbers, of a triangle's side given the measurements for two sides, and assess whether three given values can be used to form a triangle. They will create and solve inequalities to represent their conclusions.
ELD Objective(s): By the end of this lesson, students will be able to justify why three values can or cannot be used to form a triangle and why certain values must be the minimum and maximum lengths of a triangle using clear, concise sentences.

Lesson Resources: Ruler, notebook, pencil, whiteboard/markers, index cards, *Triangle Inequality Theorem Practice 1*
Room Set-Up: Students sit in pairs of desks that are arranged in four rows, facing the whiteboard. The camera is situated in the back of the room, facing the whiteboard. There are no assigned seats.

Student Needs

Student Need	Supports
Underperforming Student	Frequent one-on-one check ins, equity sticks to ensure participation, opportunities to provide written feedback to teacher (exit card)
RFEP Learners (formerly longer term ELs)	Translated instructions and materials, opportunities for peer discussions, provide written and oral definitions of vocabulary/mathematical symbols
GATE Learners	Ask high-level DOK questions, provide opportunities to demonstrate problem-solving skills to peers, provide rigorous and challenging practice problems

Academic Language

Academic	Discipline-Specific
Maximum, Minimum, Distance, Point, Length, Form	Right angle, Midpoint, Equilateral Triangle, Line segment, Triangle Inequality Theorem, Inequality symbols (< , >)

Identified Language Demand	Support
Function: *Justify* why three given values can or cannot form a triangle and why certain values must be the minimum and maximum whole number lengths of a triangle using the Triangle Inequality Theorem	Provide sentence frames (see *Triangle Inequality Theorem Practice 1*), model during instruction time, facilitate student demonstrations and explanations, multiple means of engagement and assessment (writing, drawing diagrams)
Mathematical Precision & Syntax: *Compare* the lengths of a triangle's sides using inequality symbols	Frequently translate between words and symbols (ex. "Greater than", >), Provide mnemonic device
Vocabulary: Distance, Point, Length	Utilize real life examples, provide visual representations.
Discourse: *Explain* why the Triangle Inequality Theorem must be true	Turn and talk (Section: *Conceptual Understanding*)

Informal Assessments:
Verbal student responses to questions
Student whiteboard demonstrations
Observations during *Independent Work*

Formal Assessments:
Exit card

1. Explain how you know the Triangle Inequality Theorem is true in your own words. You can use pictures, graphs, and diagrams to illustrate your explanation.
2. This lesson was: (too fast/too slow/just right)

Section/Time	Teacher Will/ Lesson Description	Students Will:
Materials (5 min)	Instruct students to get out their notebooks and a pencil. Have students obtain rulers.	Retrieve materials
Exploration (15 min)	1. Instruct students to draw a triangle with sides lengths of 3in. Have students give me a "thumbs up" when done. Ask a student to explain how he/she drew the triangle, then draw triangle on board. Explain that an equilateral triangle can be drawn by finding midpoint of the first segment and joining two sides above midpoint.	1. Attempt to draw triangle notebook. Explain process of how they drew the triangle. 2. Attempt to draw triangle. Explain process of how they

	2. Instruct students to draw a triangle with lengths of 3, 4, and 5in. Have students give me a "thumbs up" when they are done. Ask a student to explain how he/she drew the triangle. Explain that this triangle can be formed by creating a right angle, drawing the 3in and 4in sides along the right angle, and connecting the two sides with a 5in side. 3. Instruct students to draw a triangle that has side lengths of 5, 2, and 1in. Have students raise their hand if they successfully drew this triangle. Check the measurements of any student that raises his/her hand. Conclude that it is not possible to draw a triangle with these lengths.	drew their triangle. 3. Draw figure with three sides that don't connect or draw a triangle that does not have the side lengths instructed. Conclude that it is not possible to draw a triangle with side lengths of 5, 2, and 1in.
Conceptual Understanding (20 min)	1. Write *"How do I know if I can form a triangle with lengths a, b, and c?"* on the whiteboard. Write measurements for three previous exercises and assign shortest value *a*, longest value c, and remaining value *b*. Instruct students to share with a partner what they notice is different about the third set of values (Turn & Talk). Call on three random groups to share with the whole class. 2. Draw two points on the board, labeled *[school name]* and *Boba Time*. Connect two points with line segment labeled *7th street*. Ask *"Can we draw two straight lines that connect [school name] and Boba Time that, when added together, are shorter than 7th street?"* Ask students to discuss with partner if this is possible, and if not, why (Turn & Talk). Call on three random groups to share with the whole class. 3. Draw a point above *7th street* labeled *KBBQ*. Form a triangle between *[school name]*, *Boba Time*, and *KBBQ*. Label sides as Paths A, B, and C (replace 7th street with Path C). Ask random students for the shortest distance from *[school name]* to *Boba Time* (Path C). Ask students how they would get from *[school name]* to *Boba Time* if Path C was closed (Path A, then Path B). Create inequality: *Path A + Path B > Path C*. Repeat for Paths A and B. 4. Remind students that "$>$" symbol means "greater than". Remind students to put bigger number on "mouth" side of symbol (the alligator eats the bigger number).	1. Discuss with partner the differences between the three sets of numbers. 2. Discuss with partner whether it is possible to draw two lines that connect two points that are shorter than one line. Explain that the shortest distance between two points will always be one straight line, and that a shorter or equal distance cannot be created with two straight lines. 3. Explain that if Path C was closed, one would have to take Path A, then Path B. Observe that Path A + B is longer than Path C. Copy inequalities into notebook.
Procedural Fluency (15-20 min)	1. Using triangle from previous section, set a = 7, b = 6, and c = 3 Write three inequalities that prove that lengths can create triangle.	1. Copy lengths of a, b, and c in notebook along with inequalities.

	2. Set a = 3, b = 6, and c = 3. Randomly call on students to create three inequalities. Ask students if a, b, and c can create triangle (no). Remind students that sum must be greater than the third value, not equal to. 3. Ask students for "thumbs up" if they want a third example. If so, set a = 5, b = 4, and c = 2. Repeat procedure from (2).	2. Copy lengths of a, b, and c in notebook. Answer teacher's questions on how to create inequalities. Conclude that it is not possible to make a triangle with given lengths. 3. Show "thumbs up" if they would like another example.
Problem-Solving Skills (10-15 min)	1. Set a = 7 and b = 10. Ask students for maximum whole-number value of c. Ask student who answers for solution process. Write *7 + 10 = 17, 17 - 1 = 16*, and *17 > 16* on the whiteboard. Write "*the maximum value for side c is 16 units*". For minimum value, repeat process and write *7 + ? > 10, 7 + 3 = 10, 7 + 4 > 10*. 2. Set a = 4 and b = 5. Have students discuss with partner how to find maximum value. Randomly call in student for maximum value. If student doesn't know, provide hint *4 + 5 = 9*. Repeat for minimum value. 3. Ask students for "thumbs up" for additional example. If so, set a = 15, b = 12, and c = 9.	1. Write lengths of a and b in notebook. Determine that maximum length can be calculated by subtracting 1 by the sum of 7 and 10. 2. Discuss with partner how to find maximum value. Copy answers into notebook. 3. Show "thumbs up" if they need another example.
Informal Assessment: Independent Work (10 min)	Pass out *Triangle Inequality Theorem 1*. Read worksheet instructions aloud. Walk around the room and check on students, answering necessary questions.	Read worksheet instructions. Start on worksheet. Ask teacher and other students if confused.
Informal Assessment: Student Demonstrations (15-20 min)	1. Write problems 1, 2, and 3 on board. Invite students to come up to board and solve for super credit. Check answers for correctness. 2. Repeat process for problems 11, 12, and 13. 3. If time allows, invite students to solve 19-21 on whiteboard. If no one can solve these, choose one to go over.	Solve problems on whiteboard or copy solution into notebook.
Formal Assessment: Exit Card (5 min)	Pass out index cards. Sketch template of exit card on whiteboard and explain instructions. Collect responses as class ends.	Complete exit ticket and give card to teacher when finished.

Lesson Plan Two

Discipline: Mathematics - Geometry	Subject Matter Focus: Triangle Theorems
Model of Teaching: Guided Discovery	Mathematical Practices: Look for and express regularity in repeated reasoning (8), make sense of problems and persevere in solving them (1)

Central Focus: The purpose of this learning segment is for students to build upon their knowledge of geometric theorems involving triangles. They will prove these theorems by constructing triangles using rulers and protractors, building and solving inequalities, and summarizing their conclusions with formal syntax and vocabulary. They will apply these theorems to solve mathematical problems and real-world situations.
Essential Question: How can we use the properties of triangles to solve real-world problems?

CCSS Standard(s): CCSSM.G-CO.10
Prove Geometric Theorems: Prove theorems about triangles.
ELD Standard(s): CCSS.ELD.9-10.S2.P1c.BR10a
Writing: Write clear and coherent summaries of texts and experiences by using complete and concise sentences and key words (e.g., from notes or graphic organizers)

Objective(s): By the end of this lesson, students will be able to prove whether a triangle, given the side lengths, is acute, right, or obtuse, using the Converse Pythagorean Theorem.
ELD Objective(s): By the end of this lesson students will be able to explain how to use the Converse Pythagorean Theorem to classify triangles as acute, right, or obtuse using clear, concise sentences.

Lesson Resources: Notebook, pencil, protractor, ruler, calculator, whiteboard/markers, *Classifying Triangles*, *Types of Triangles Graphic Organizer*
Room Set-Up: See **Lesson One**

Student Needs

Student Need	Supports
Underperforming student	Frequent one-on-one check ins, equity sticks to ensure participation, opportunities to provide feedback to teacher (exit card)
RFEP Learners (formerly longer term ELs)	Translated instructions and materials, opportunities for peer discussions, provide written and oral definitions of vocabulary/mathematical symbols
Students struggling with basic mathematical skills (5th grade or below)	Review how to use calculator to square numbers and how to use a protractor to construct angles, provide graphic organizer and connect to real-life examples (see: *Types of Triangles Graphic Organizer*), use routine problem-solving strategy (WDIK, WDINTK, HCIST) to solve word problems (see: *Strategic Competence/Informal Assessment*)

GATE Learners	Ask high-level DOK questions, provide opportunities to demonstrate problem-solving skills to peers, provide rigorous and challenging practice problems

Academic Language
Vocabulary:

Academic	Discipline-Specific
Inequality/Inequalities Equation Compatible/Compatibility Sum	Converse Pythagorean Theorem Acute/Right/Obtuse Triangle/Angle Equilateral/Isosceles/Scalene Triangle Square/Square Root/Squared Symbols (^, <, >, ^{2})

Supports:

Identified Language Demand	Support
Function: *Prove* that triangle is acute, obtuse, or right using the Converse Pythagorean Theorem	Provide sentence frames, multiple examples during instruction time, facilitate student demonstrations, discussions, and explanations
Mathematical Precision: Square and sum numbers using calculator	Review how to use calculator and definition of squared number, model during instruction time
Vocabulary: Acute/Right/Obtuse, Equilateral/Isosceles/Scalene Triangles	Review types of triangles, provide graphic organizer, connect to real-life examples
Syntax: Comprehend and use *inequality symbols* and equations to represent the Converse Pythagorean Theorem and how it relates the sides of a triangle to one another	Model during instruction time, frequently translate between words and symbols (ex. “Greater than”, >)
Discourse: Explain how to use Converse Pythagorean Theorem to classify triangles	Turn and Talk (section: *Conceptual Understanding/ Procedural Fluency*)

Informal Assessments:
Answers to *Review* problems
Verbal student responses to questions
Observations during *Independent Work*
Student whiteboard demonstrations

Formal Assessments:
Exit Card:
1. Explain how we can classify a triangle as acute, right, or obtuse using a^2, b^2, and c^2
2. Today’s lesson was: (easy/ hard/in the middle)

Section/Time	**Teacher Will/ Lesson Description**	**Students Will:**
Review (20 min)	1. Present two problems from *Triangle Inequality Theorem 1* (Lesson One), and have students solve in notebook. Check notebooks. 2. Ask students for six types of triangles. Present graphic organizer on whiteboard and have students identify real-life examples of triangles.	1. Complete two problems in notebook. 2. Identify the six types of triangles, name real-life examples of each triangle, and complete graphic organizer in notebook.
Materials (5 min)	Instruct students to retrieve protractors, rulers, and calculators.	Retrieve materials
Exploration: Conceptual Understanding/ Procedural Fluency (20-25 min)	1. Assign students a number from 1 to 3. Tell 1s to draw acute triangle in notebook, 2s to draw right, and 3s to draw obtuse. Review how to use protractor to sketch angles. Walk around room and check triangles for correctness. 2. Tell students to measure sides of their triangle (in centimeters, rounding to nearest half cm) and label the shortest side *a*, the medium side *b*, and the longest side *c*. 3. Ask students to square the measurements of a, b, and c, then add the squares a and b. Remind students how to use calculator to find squares. 4. Have students hit their desks if their sum of squares of a and b was bigger than square of c. Repeat for equals (or approximately equal to) and less than c squared. 5. Ask students to discuss observations with a partner (Turn & Talk). Call on a few pairs and ask what they discussed, recording answers on whiteboard. 6. Write inequalities on board for	1. Draw assigned triangle in notebook using protractor. 2. Label and measure sides of triangle in cm. 3. Use calculator to find squares of triangle sides. Add squares of a and b. 4. Students who were assigned a 1 should hit their desks. Students who were assigned a 2 should hit their desks when teacher asks if a squared and b squared equals c squared. Students assigned a 3 should hit their desks when teacher asks if sum of squared was bigger than c squared. 5. Discuss observations with partner. Observe that students who drew acute triangle had sum larger than c squared, students who drew right triangle had sum equal to c squared, and students who drew obtuse triangle had sums smaller than c squared. 6. Copy inequalities into notebook.

	acute, right, and obtuse triangles.	
Procedural Fluency/Informal Assessment (20-25 minutes)	Pass out *Classifying Triangles*. Walk around the room and answer students' questions. Invite students to demonstrate problems 1-8 on the whiteboard for super credit.	Beginning working on *Classifying Triangles*. Ask teacher/other students questions if confused. Demonstrate solutions to problems 1-8 on whiteboard or copy solutions into notebook
Strategic Competence/Informal Assessment (25 min)	1. Call on random students to choose one of problems 9-15. Use WDIK, WDINTK, HCIST (*What Do I Know, What Do I Need To Know, How Can I Solve This*) strategy to set up equations. Call on random students to answer WDIK and WDINTK questions. 2. Ask if any student was able to solve problem. Invite student to come up to whiteboard and demonstrate solution, or solve solution and compare answer with student(s).	1.(Problem selection dependent) Identify that two of the side lengths of the triangle are given. Use side lengths of triangle and Converse Pythagorean Theorem to either calculate missing side length or classify triangle as right, acute, or obtuse. 2. If attempted, explain answer and process for getting answer. Demonstrate solution on whiteboard, with teacher guidance if necessary, for super credit and/or ask questions about process for solving problem.
Formal Assessment (5 min)	Pass out index cards. Sketch template of exit card on whiteboard and explain instructions. Collect responses as class ends.	Complete exit ticket and give card to teacher when finished.

Lesson Plan Three

Discipline: Mathematics - Geometry	Subject Matter Focus: Triangle Theorems
Model of Teaching: Group Investigation	Mathematical Practices: Construct viable arguments and critique the reasoning of others (3), Use appropriate tools strategically (5)

Central Focus: The purpose of this learning segment is for students to build upon their knowledge of geometric theorems involving triangles. They will prove these theorems by constructing triangles using appropriate tools, building and solving equations with inequalities, and summarizing their conclusions using formal syntax and vocabulary. They will apply these theorems to solve mathematical problems and real-world situations.
Essential Question: How can we use the properties of triangles to solve real-world problems?

CCSS Standard(s): CCSSM.G-CO.10
Prove Geometric Theorems: Prove theorems about triangles
ELD Standard(s): CCSS.ELD.9-10.S2.P1c.BR10a
Writing: Write clear and coherent summaries of texts and experiences by using complete and concise sentences and key words (e.g., from notes or graphic organizers)

Objective(s): By the end of this lesson, students will be able to use Triangle Congruence Theorems to analyze a pair of triangles and determine whether congruence can be established based on the given sides and angles. They will apply their knowledge of Triangle Congruence Theorems to real-world situations.
ELD Objective(s): By the end of this lesson, students will be able to prove whether two triangles are congruent and summarize their conclusions using clear, concise sentences.

Lesson Resources: Notebook, pencil, whiteboard/markers, scissors, protractor, ruler, *Puzzlement Problem*, *Resources*, *Congruent Triangles Activity*, *Triangle Congruence Graphic Organizer*, *Triangle Congruence Practice*
Room Set-Up: See **Lesson One**

Student Needs

Student Need	Supports
Underperforming student	Frequent one-on-one check ins, equity sticks to ensure participation, opportunities to provide feedback to teacher (exit card)
RFEP Learners (Formerly Long-Term ELs)	Simple, one-step instructions, opportunities for peer discussion, variety of assessment methods, explicitly teach/review vocabulary, provide sentence frames (*Puzzlement Problem*)
Struggling with basic reading skills (5th grade or below)	Chunked instructions, provide visuals to accompany instructions, bold key words (*Congruent Triangles Activity*)
Struggling with basic	Review corresponding sides and angles of geometric figures,

mathematics skills (5th grade or below)	congruence symbols/notation, and how to use protractor. One on one assistance with measuring using a ruler. Provide graphic organizer (*Triangle Congruence Graphic Organizer)*
GATE Learners	Ask high-level DOK questions, provide opportunities to demonstrate problem-solving skills to peers, provide rigorous and challenging practice problems

Academic Language
Vocabulary:

Academic	Discipline-Specific
Congruence/Congruent, Corresponding, Degrees (of an angle)	Acronyms (SSS, ASA, SAS, AAS), Vertical Angles, Included Angle/Side, Triangle Congruence Theorems, Symbols (ticks, ≅)

Supports:

Identified Language Demand	Support
Function: *Prove* that two triangles are congruent or not congruent using Triangle Congruence Theorems	Provide hands-on materials, multiple representations, verification through experimental methods, scaffolded questioning
Vocabulary: Describe and identify vertical angles, use acronyms in appropriate context	Review vocabulary and provide examples, provide graphic organizers
Symbols: Comprehend and use congruence symbols to identify congruent sides/angles of two triangles	Review symbols and model examples
Mathematical Precision: Distinguish between *included* and non-included sides/angles	Explicitly teach concept, model examples (section: *Discussion*)
Discourse: *Explain* the meaning of Triangle Congruence Acronyms (SSS, ASA, SAS, AAS)	Opportunities for both peer and whole-class discussions, multiple means of engagement (writing, talking, constructing figures)

Informal Assessments:
Answers to *Review* problems
Student responses in *Discussion* section
Observations during *Practice/Procedural Fluency* question

Formal Assessments:
Lesson Three: *Puzzlement Problem*

Exit card
1. Why does ASA prove that two triangles are congruent, but SSA does not?
2. Please rate your understanding of this lesson from a scale of 1-4

Section/Time	**Teacher Will/ Lesson Description**	**Students Will:**
Review (20 min)	Present one problem from *Classifying Triangles* and one problem from *Triangle Inequality Theorem Practice 1* on whiteboard. Have students complete in notebook. Use WDIK, WDINTK, HCIST (*What Do I Know, What Do I Need To Know, How Can I Solve This*) strategy to solve problems with whole class.	Complete assigned problems in notebook
Puzzlement/Problem Solving Skills (5 min)	Present *Puzzlement Problem*. Have students discuss solution with partner (Turn & Talk) and write initial answer. Randomly call on a few pairs to share what they discussed.	Discuss *Puzzlement Problem* with partner and write initial answer.
Review (10 min)	Review definition of congruent triangles (all corresponding sides and angles are the identical in measurement). Model example of two congruent triangles with appropriate symbols. Present lesson question: *how can we tell if two triangles are congruent if we aren't given all the sides and angles?*	Copy down example of congruent triangles in notebook (if they don't remember)
Materials & Activity Introduction (10 min)	1. Instruct students to work in pairs and retrieve scissors. Pass out *Lesson Three Resources*, and instruct Partner A to cut out the figures on page one, and Partner B to cut out the figures on page two. Pass out *Congruent Triangles Activity*, and randomly select students to read instructions aloud. 2. Instruct one half of room to complete parts 1-3 of activity, and other half to complete parts 4-6. Instruct students that they will have 45 minutes to complete activity.	1. Find a partner to complete activity with and retrieve materials. Decide who is Partner A and Partner B. Cut out figures on one page of *Lesson Three Resources*. 2. Read instructions for and begin assigned parts of *Congruent Triangles Activity*.
Activity/Conceptual Understanding (45 min)	Walk around classroom, answer student questions, and assist students on *Congruent Triangles Activity*. Draw chart on whiteboard with six sections, one for each part of activity. Invite students to write answers on whiteboard for super credit.	Work on *Congruent Triangles Activity* with partner. Sketch triangles and answer questions in notebook. Draw chart on whiteboard with a section for each part of activity.

		Invite students to write answers on whiteboard for super credit.
Discussion (20 min)	1. Ask first half of class to describe task and share results from parts 1-3. Repeat for second half of class. 2. Distinguish between *included* and *non-included* sides/angles. Complete *Triangle Congruence Graphic Organizer*	1. Share findings from *Congruent Triangles Activity* with class. 2. Complete *Triangle Congruence Graphic Organizer*
Practice/Procedural Fluency (10 min)	Pass out *Triangle Congruence Practice*. Model two problems on whiteboard. Review definition of vertical angles and present examples.	Complete *Triangle Congruence Practice* in notebook. Copy definition and examples of vertical angles into notebook (if they don't remember)
Formal Assessments: Puzzlement Problem & Exit Card/Problem Solving Skills (10 min)	1. Instruct students to write final answer to *Lesson Three: Puzzlement Problem* and collect papers once finished. 2. Pass out index cards. Sketch template of exit card on whiteboard and explain instructions. Collect responses as class ends.	1. Write final answer to *Puzzlement Problem*. 2. Complete exit ticket and give card to teacher when finished.